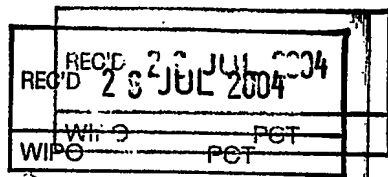




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4. Title of the invention

METHOD AND APPARATUS FOR FRAGILE WATERMARKING

5. Name of your agent *(if you have one)**"Address for service" in the United Kingdom to which all correspondence should be sent (including the postcode)*McCORMACK, DEREK JAMES  
EUROPEAN INTELLECTUAL PROPERTY DEPARTMENT  
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ALENCON LINK  
BASINGSTOKE  
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D. McCormack

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Method and Apparatus for Fragile WatermarkingTechnical Field

5 The invention relates to a method and apparatus for fragile watermarking, and in particular a method and apparatus for fragile watermarking and a method for validating such a fragile watermark.

10 Background

Photographs, paintings, film material and other artistic works have for many years been recorded and transmitted using analogue carriers. However, their reproduction and processing is time consuming, involves a heavy workload and  
15 lead to degradation of the original material. This means that content produced and stored using analogue devices has an in-built protection against unintentional changes and malicious manipulation. In general, deliberate changes in analogue media are not only difficult but can easily be  
20 perceived by a human inspector.

Recently however, digital media have become pervasive, and threaten to completely substitute their analogue counterparts. Furthermore, affordable media processing  
25 tools and fast transmission mechanisms are ubiquitous. As a consequence digital content can nowadays be accurately copied, processed and distributed around the world within seconds. Creators, legitimate distributors and end-users enjoy the flexibility and user friendliness of digital  
30 processing tools and networks to copy, process and distribute their content over open digital channels at high speed. However, they also need to guarantee that material used or being published at the end of the distribution chain is genuine. Consequently, automatic tools to  
35 establish the authenticity and integrity of digital media are highly important.

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Secure communications problems have largely found a solution in cryptography, which guarantees message integrity by using digital signatures with secret keys. However, traditional cryptosystems do not permanently

- 5 associate cryptographic information with the content. Cryptographic techniques do not embed information directly into the message itself, but rather hide a message during communication.

- 10 To provide security by using signatures embedded directly in the content, additional methods need to be considered. Techniques that have been proposed to address this problem belong to a more general class of methods known as digital watermarking, as for example may be found in Signal
- 15 Processing, Special Issue on Watermarking, vol. 66, no. 3 May 1998.

- Several watermarking schemes that address image authentication have been previously developed and fall into
- 20 two basic categories: fragile and semi-fragile.

- Fragile watermarking schemes address the detection of any image changes. Semi-fragile watermarking schemes are designed to discriminate between expected image changes, in
- 25 most cases due to application constraints, e.g., compression to meet bandwidth requirements, and intentional image tampering.

- In the case of fragile watermarking, a number of schemes
- 30 exist in the prior art:

One prior scheme is proposed in S. Walton, "Information Authentication for a Slippery New Age", Dr. Dobbs Journal, vol. 20, no. 4, Apr. 1995, pp. 18-26. The scheme uses a

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check-sum built from the 7 most significant bits of a given pixel, which is then inserted as the least significant bit of the pixel. However, the watermark has only limited security, primarily due to the ease of calculating new  
5 check-sums.

Another prior fragile watermarking scheme is proposed in M. M. Yeung and F. Mintzer, "An Invisible Watermarking Technique for Image Verification", Proc. ICIP, Santa  
10 Barbara, California, 1997. The Yeung-Mintzer algorithm uses a secret key to generate a unique mapping that randomly assigns a binary value to grey levels of the image. This mapping is used to insert a binary logo or signature in the pixel values. Image integrity is inspected by direct  
15 comparison between the inserted logo or signature and the decoded binary image. The main advantage of this algorithm is its high localization accuracy derived from the fact that each pixel is individually watermarked. However, the Yeung-Mintzer algorithm is vulnerable to simple attacks as  
20 shown in J. Fridrich, "Security of Fragile Authentication Watermarks with localization", Proc. SPIE, vol. 4675, No. 75, Jan. 2002.

A third prior scheme for image authentication is proposed  
25 in P. W. Wong, "A Public Key Watermark for Image Verification and Authentication", Proc. ICIP, Chicago, Illinois, Oct. 1998. This scheme embeds a digital signature extracted from the most significant bits of a block of the image into the least significant bit of the pixels in the  
30 same block. However this scheme was shown to be vulnerable to a counterfeiting attack in M. Holliman and N. Memon, "Counterfeiting Attacks on Oblivious Block-Wise Independent Invisible Watermarking Schemes", Proc. IEEE Trans. on Image Processing, vol 9, no 3, Mar. 2000, pp. 432-441. This

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attack belongs to the class of vector quantisation counterfeiting and has been shown to defeat any fragile watermarking scheme that achieves localization accuracy by watermarking small independent image blocks.

5

One common feature of these and other prior schemes from the literature is that authentication signatures are embedded in the image content, either in the pixel or a transform domain, and the security of the schemes resides in a hash or encryption mechanism.

10

This ultimately leaves such schemes vulnerable to the attacks noted above.

15

Thus there is a need for an alternative method of fragile watermarking.

The purpose of the present invention is to address the above problem.

20

#### Summary of the Invention

The present invention provides a method of fragile watermarking, characterised by the step of generating at least a first ill-conditioned operator, said ill-conditioned operator being related to values extracted from an image or portion thereof A.

25

In a first aspect, the present invention provides a method of fragile watermarking, as claimed in claim 1.

30

In a second aspect, the present invention provides a method of verifying a fragile watermark, as claimed in claim 19.

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In a third aspect, the present invention provides apparatus for fragile watermarking, as claimed in claim 23.

5 In a fourth aspect, the present invention provides apparatus for verifying a fragile watermark, as claimed in claim 24.

Further features of the present invention are as defined in the dependent claims.

10

Embodiments of the present invention will now be described by way of example with reference to the accompanying drawings, in which:

15 Brief description of the drawings

FIG. 1 is a block diagram of a method of fragile watermarking in accordance with an embodiment of the present invention.

20

FIG. 2 is a block diagram of a method of verifying a fragile watermark in accordance with an embodiment of the present invention.



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Detailed description

Referring to FIGs. 1 and 2, a method of fragile watermarking 100 and a method of validating said fragile watermark 200 are disclosed. In the following description, a number of specific details are presented in order to provide a thorough understanding of the present invention. It will be obvious, however, to a person skilled in the art that these specific details need not be employed to practice the present invention. In other instances, well known methods, procedures and components have not been described in detail in order to avoid unnecessarily obscuring the present invention.

## 1. Fragile watermarking.

- 15 An embodiment of the present invention provides a method providing an essentially different approach from those reported in the prior art. This method is based on the inherent instability property of inverse ill-conditioned problems, and the fact that small changes to their input data cause large changes in any approximate solution. Singular valued decomposition and other fundamental linear algebra tools are used to construct an ill-conditioned matrix interrelating the original image and the watermark pattern. This is achieved by exploiting the relation between singular values, the least square solution of linear algebraic equations and the high instability of linear ill-conditioned operators.
- 30 In brief, an embodiment of the present invention performs a fragile watermarking method on blocks (portions) of pixels extracted from the image to be watermarked, the values in this block being treated as a matrix for the purposes of analysis.

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Similarly, blocks of pixels of corresponding size are taken from a watermark pattern, or are generated to resemble such blocks.

5

The smallest singular value of the matrix to be watermarked is replaced to artificially create an ill-conditioned minimization problem. The solution to this problem involves a least squares approximation of the previously defined  
10 ill-conditioned operator in order to find an unknown parameter.

This solution process links the watermark with the image using the underlying ill-conditioned operator. An image  
15 block is considered watermarked by setting its smallest singular value equal to a parameter estimated from the minimization task.

Thus, the watermark is spread over the whole image in a  
20 subtle but quite complex manner. One advantage of this method is that the distortion induced by the watermarking procedure can be strictly controlled since it depends only on changes in the smallest singular values of each block.

25 The verification procedure solves the same optimisation problem and compares the norm of the solution with a large secret number  $N$  used in the generation of the watermark.

Any small change to the image will therefore result in the  
30 norm of the solution differing significantly from the large secret number  $N$ , due to the property of the ill conditioned operator to produce highly differing solutions in response to small changes in the input values.

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The method will now be described in detail;

Firstly, to illustrate how an ill-conditioned operator may be used to provide a watermark, consider that in many known applications of linear algebra, it is necessary to find a good approximation  $\hat{x}$  of an unknown vector  $x \in \mathbb{R}^n$  satisfying the linear equation

$$Bx = b, \quad \text{Eq. 1}$$

for a given right-hand side vector  $b \in \mathbb{R}^n$ . The degree of difficulty in solving Eq. 1 depends upon the condition number of the matrix  $B$ . The vector  $\hat{x} = B^+b$  would seem to be a solution of Eq. 1, where,  $B^+ = (B^TB)^{-1}B^T$ , i.e.,  $B^+$  denotes the pseudo-inverse of  $B$ .

However, if  $B$  is ill-conditioned or singular then  $\hat{x} = B^+b$ , if it exist at all, is a poor approximation of  $x$ .

An error estimate given by  $\|x - \hat{x}\| \leq \|B^+\| \|B\hat{x} - b\|$  shows that the approximation error can grow proportional to the norm of the inverse of  $B$ . Since the norm of the inverse of  $B$  is proportional to the condition number of  $B$ , it is evident that the more the ill-conditioning of  $B$ , the larger the difference between  $x$  and  $\hat{x}$ .

Furthermore, the estimation of the inverse of an ill-conditioned matrix is not straightforward, and clearly is essentially the same problem as seen in Eq. 1. Moreover, when  $B$  is ill-conditioned, solving Eq. 1 becomes equivalent to solving the optimisation problem  $\min_{x \in \mathbb{R}^n} \|Bx - b\|^2$  for a

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predefined norm  $\|\cdot\|$ . It is well-known in the art that the  $L_2$ -norm solution of this least squares problem is given by

$$\hat{x} = \sum_{s_i(B) \neq 0} \frac{u_{A_i}^T b}{s_i(B)} v_i. \quad \text{Eq. 2}$$

It becomes evident from Eq. 2 that errors in either any of the left singular vectors of  $B$  or in the right-hand side  $b$  are drastically magnified by the smallest of the singular values  $s_i(B)$  of  $B$ .

In an embodiment of the present invention, an ill-conditioned operator  $B$  is generated that is related to values extracted from an image or a portion thereof  $A$ . In this manner alterations to  $A$  will be reflected in magnified errors to a solution for  $\hat{x}$  of similar form to Eq. 2.

In an embodiment of the present invention, a watermarking pattern  $\Omega$  is interrelated with an image  $I$ , thus watermarking it, as follows:

Given an image  $I$  of dimensions  $m \times n$ , a watermark pattern  $\Omega$  of typically the same dimensions is built.

In a first embodiment,  $\Omega$  is an array of pseudo-randomly generated binary or real numbers.

In an alternative embodiment of the present invention, the procedure to generate  $\Omega$  uses a single or repeated instance of a logo, typically a binary pattern, combined with pseudo-randomly generated numbers; initially, a mosaic-like binary image  $P$  of dimension  $m \times n$  is built by tiling the logo to occupy an area similar to the original image  $I$ . The watermark pattern is then typically defined as  $\Omega = P \oplus w$ ,

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where  $w$  is a  $m \times n$  array of pseudo-randomly generated binary numbers and  $\oplus$  denotes the bitwise XOR operator.

5 In either embodiment, no assumption needs to be imposed on the statistical properties of the random number generator; the binary or real numbers used to generate the watermark can follow any probability distribution and are not restricted to Gaussian or uniform. This is because the present invention does not rely on statistical analysis for  
10 authentication or tamper detection. However it will be clear to a person skilled in the art that in consequence statistical constraints can be imposed if desired.

In either embodiment,  $\Omega$  depends on a secret key  $K$  whose  
15 value seeds the pseudo-random number generator.  $K$  is subsequently also used in validating the watermark.

In an embodiment of the present invention, the watermarking process is performed in a block-wise fashion. For the sake  
20 of simplicity and without loss of generality, for the following description assume that an image  $I$  is partitioned into  $L$  small blocks or portions  $A^{(k)}$ ,  $k = 1, \dots, L$  of dimensions  $p \times q$ . Likewise,  $\Omega$  is partitioned into  $L$  blocks or portions  $W^{(k)}$ ,  $k = 1, \dots, L$  of dimension  $p \times q$ . For the  
25 sake of notational simplicity the upper index representing the block number will be omitted hereon in unless expressly referred to. Without loss of generality for the following description assume that the blocks are square, i.e.,  
 $p = q = n$ .

30

Thus blocks  $A$  and  $W$  can be considered for the purpose of explanation to be generic matrices comprising values

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obtained from the original image and watermark, respectively.

As is known in the art, a fundamental result of Linear Algebra states that matrix  $A$  can be represented as

$$A = U_A S_A V_A^T, \quad \text{Eq. 3}$$

i.e. a singular value decomposition of  $A$ , where

$U_A = (u_1, \dots, u_n) \in \mathbb{R}^{n \times n}$  and  $V_A = (v_1, \dots, v_n) \in \mathbb{R}^{n \times n}$ . The columns  $\{u_k\}$ ,  $k = 1, \dots, n$  of  $U_A$  are called the left singular

vectors and form an orthonormal basis, i.e.,

$u_i \cdot u_j = 1$ , if  $i = j$  and  $u_i \cdot u_j = 0$  otherwise. The rows of  $V_A^T$

are the right singular vectors,  $\{v_k\}$ ,  $k = 1, \dots, n$  and also

form an orthonormal basis.  $S_A = \text{diag}(s_1(A), \dots, s_n(A))$  is a diagonal matrix whose diagonal elements are the singular

values of  $A$ . If  $\text{rank}(A) = r \leq n$ , then

$s_k(A) > 0$ , for  $k = 1, \dots, r$ ,  $s_k(A) \geq s_{k+1}(A)$ , for  $k = 1, \dots, r-1$

and  $s_k(A) = 0$ , for  $k > r$ . Consequently  $s_r(A)$  is the smallest real positive singular value of  $A$  in  $S_A$ .

In an embodiment of the present invention, it is proposed to replace  $s_r(A)$  with a real positive number  $\hat{s}_r(A)$  as part of the process to produce a watermarked version  $\hat{A}$  of  $A$ , where the distortion introduced by the watermarking process is determined with reference to the calculation of

$$\|A - \hat{A}\|_2 = |s_r(A) - \hat{s}_r(A)|, \quad \text{Eq. 4}$$

where  $\|\cdot\|_2$  denotes the  $L_2$ -norm, and thus the distortion is dependent upon the value of  $\hat{s}_r(A)$ .

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Given an image or portion thereof  $A$ , the corresponding watermarked portion or block is defined as the matrix  $\hat{A}$ , generated according to the following considerations. Observe that  $A$  and  $\hat{A}$  have the same dimensions.

5

Initially, singular value decomposition of  $A$  and  $W$  is performed to obtain  $A = U_A S_A V_A^T$  and  $W = U_W S_W V_W^T$ , respectively. Let  $S_A = \text{diag}(s_1(A), \dots, s_r(A))$  and  $S_W = \text{diag}(s_1(W), \dots, s_t(W))$  be the nonzero singular values of

10

$A$  and  $W$  respectively. The two diagonal matrices  $\hat{S}_A = \text{diag}(s_1(A), \dots, \hat{s}_r(A))$  and  $\hat{S}_W = \text{diag}(s_1(W), \dots, \hat{s}_t(W))$  are then built by replacing the last nonzero singular values  $s_r(A)$  and  $s_t(W)$  by two specific real positive numbers  $\hat{s}_r(A)$  and  $\hat{s}_t(W)$ , respectively. Here it is assumed that the

smallest nonzero singular value of  $A$  is  $s_r(A)$ , i.e.,  $\text{rank}(A) = r$  and the smallest nonzero singular value of  $W$  is  $s_t(W)$ , i.e.,  $\text{rank}(W) = t$ . Using  $\hat{S}_A$  the watermarked block  $\hat{A}$  is defined as

15

$$\hat{A} = U_A \hat{S}_A V_A^T. \quad \text{Eq. 5}$$

20

Likewise,  $\hat{S}_W$  is used to build an ill-conditioned matrix  $\hat{W}$  according to

$$\hat{W} = U_W \hat{S}_W V_W^T. \quad \text{Eq. 6}$$

25

Now, one should choose the two values  $\hat{s}_r(A)$  and  $\hat{s}_t(W)$ .

In selecting values of  $\hat{s}_r(A)$  and  $\hat{s}_t(W)$ , it is desirable to do so in such a fashion as to facilitate the fragility of the watermarking process, the uniqueness of the watermark thus

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made and optionally the control of perceptibility of the watermark in the final watermarked image  $\hat{f}$ .

#### A. Fragility.

- 5 It is desired that any change to single or multiple elements of  $\hat{A}$  can be detected by a validation procedure.

In an embodiment of the present invention, replacing  $s_r(W)$  with  $\epsilon$  in the calculation of Eq. 6 achieves this if  $\epsilon$  is a  
10 sufficiently small positive real number, increasing the condition number of the singular value matrix  $S_W$  and so making  $\hat{W}$  extremely ill-conditioned.  $\hat{A}$  and  $\hat{W}$  are then interrelated using matrix multiplication to produce the ill conditioned matrix  $B = \hat{A}\hat{W}$ .

15

Although by addressing the requirement of fragility  $\hat{W}$  is now defined using  $\epsilon$ ,  $\hat{A}$  still depends on an unknown parameter  $\hat{s}_r(A)$ . For that reason  $B$  should be regarded as a parametric family of matrices:

20

$$B(\hat{s}_r) = \hat{A}(\hat{s}_r)\hat{W}.$$

Eq. 7

This parametric family of matrices  $B(\hat{s}_r)$  110 determines the linear ill-conditioned operator used in the fragile watermarking method, and is resolved by addressing the second requirement:

25

#### B. Uniqueness.

It is desirable to select from amongst the parametric family of matrices  $B(\hat{s}_r)$  a single operator for use in a specific fragile watermark.

30



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For a pre-defined large real number  $N$ , there exists a unique value of  $\hat{s}_r(A)$ , so that the  $L_2$ -norm solution of the least squares problem

$$\min_{x \in \mathbb{R}^p} \|Bx - b\|_2^2, \quad \text{Eq. 8}$$

5 is  $N^2$ . Here,  $b$  is an arbitrary vector defining the right-hand side of the linear system to be minimized in Eq. 8.

Thus in an embodiment of the present invention, by selecting a value of  $N$  as a key, a corresponding unique  
10 value  $\bar{s}_r(A)$  can be found from the solution of Eq. 8.

By using this unique value  $\bar{s}_r(A)$  as  $\hat{s}_r(A)$  120, a watermarked image block  $\tilde{A}$  dependent both upon key  $N$  via Eq. 8 and key  $K$  via Eq. 7 is produced using  $\tilde{A} = U_A \hat{S}_A V_A^T$  130, with the  
15 watermark distributed over the entire block  $\tilde{A}$  through manipulation of the smallest singular value of  $A$ .

### C. Perceptibility.

Whilst the processes described above to address the  
20 conditions of fragility and uniqueness are sufficient to provide a watermarked block  $\tilde{A}$ , in an enhanced embodiment of the present invention the selected value of  $\hat{s}_r(A)$  is additionally constrained to lie in the interval  
 $\max(\text{eps}, s_r(A) - \delta) \leq \hat{s}_r(A) \leq s_r(A) + \delta$ , where  $\text{eps}$  is the machine  
25 precision and  $\delta$  is a scalar used to control the distortion to the image block  $A$  induced by the watermark in  $\tilde{A}$ .

The expression  $\max(\text{eps}, s_r(A) - \delta)$  ensures that  $\hat{s}_r(A)$  remains nonzero and positive. This condition together with Eq. 2

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allows the distortion to be kept below a user-defined value  $\delta$ .

In an embodiment of the present invention, the method of fragile watermarking of an image  $I$  comprises the following steps:

- i. Generating a  $K$ -dependent watermark pattern matrix  $W$  from  $\Omega$ , or recalling a pre-existing one;
- 10 ii. Constructing the parametric family of matrices  $B(\hat{S}_r)$  as defined by Eq. 7.
- iii. Estimating the unique parameter  $\bar{S}_r(A)$ , that minimizes the expression:

$$\min_{\hat{S}_r} \left\{ \sum_{i=1}^q (u_{B_i}^T b / s_i(B(\hat{S}_r)))^2 - N^2 \right\}, \quad \text{Eq. 9}$$

15 (based on Eq. 2) where  $u_{B_i}$  is the  $i$ -th column of the matrix formed with the right singular vectors of  $B$ ,  $s_i(B)$  are the singular values of  $B$ ,  $b$  is the right-hand side vector given in Eq. 8 and key  $N$  is a large real number.

- 20 iv. Estimating the watermarked block  $\hat{A} = U_A \hat{S}_A V_A^T$  by setting  $\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{S}_r(A))$ .

In an otherwise similar enhanced embodiment of the present invention, step iii. above comprises estimating the unique parameter  $\bar{S}_r(A) \in [\max(\text{eps}, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$ , that minimizes the expression:

$$\min_{\hat{S}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^q (u_{B_i}^T b / s_i(B(\hat{S}_r)))^2 - N^2 \right\}, \quad \text{Eq. 10}$$

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In both the directly preceding embodiments, step iv. shows how the value  $\hat{s}_r(A)$  in Eq. 5 is chosen, namely by setting  $\hat{s}_r(A) = \bar{s}_r(A)$ , where  $\bar{s}_r(A)$  is the result of the minimization problem of Eq. 9 or 10. Like  $K$ , the number  $N$  in Eq. 9 or 10 is also secret. Although it is possible to select a value of  $N$  dependant on  $K$  or vice-versa, higher security is achieved when  $N$  and  $K$  are chosen independently. Thus, the security of the proposed approach resides in the secrecy of set of keys  $\kappa = \{K, N\}$ .

In an enhanced embodiment of the present invention, the value of  $b$  selected for equations 8, 9 or 10 is made dependant upon a parameter derived from a portion of image  $I$  other than current portion  $A$ :

For a sequential watermarking process comprising the watermarking of portion  $A^{(k)}$  after the watermarking of portion  $A^{(k-1)}$ , for  $k=1, \dots, L$  of  $L$  portions of image  $I$ , then the step of calculating  $b^{(k)}$  for portion  $A^{(k)}$  comprises calculating substantially the following equation part:

$$b^{(k)} = \begin{cases} A^{(k)} Z^{(k)} & \text{for } k = 1 \\ A^{(k-1)} Z^{(k)} & \text{else} \end{cases} \quad \text{Eq. 11}$$

where  $Z^{(k)}$  is a pseudo-random binary vector.

This enhancement increases the difficulty of successfully undertaking a vector quantisation attack upon the image  $I$ , requiring that larger image areas containing several authenticated blocks are replaced. Even then, the blocks at the border of the swapped area will be declared faked:

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To validate authenticity and to detect tampered areas, a receiver of a received image  $I^*$  needs to test if the received image or a portion thereof  $A^*$  has been tampered with or not. It is assumed that the receiver is a trusted party who knows the secret set of keys  $\kappa = \{K, N\}$ .

In an embodiment of the present invention, in addition to  $\epsilon$  a tolerance value  $\tau$  is used in the verification process. This parameter provides tolerance to approximation errors inherent to any numerical process.  $\epsilon$  and  $\tau$  are fixed numbers and so can be known to the public. Referring to FIG. 2, most steps of the verification procedure coincide with the steps of the watermarking procedure:

Using  $K$ , the receiver first generates the watermark pattern or portion thereof  $W$ . Next,  $\epsilon$  is used to build the matrix  $\hat{W}$  by setting  $\hat{S}_w = \text{diag}(s_1(W), \dots, \epsilon)$  as in Eq. 6.

Afterwards, the ill-conditioned matrix  $B^* = A^* \hat{W}$  is built and the solution of the minimization problem

$$\min_{x \in \mathbb{R}^p} \|B^* x - b\|_2^2 \quad \text{Eq. 12}$$

is calculated. Once Eq. 12 has been solved  $N^*$  is defined as the square root of the norm of the vector  $x$  minimizing Eq. 12.

The verification step consists of a comparison between  $N^*$  and the secret value  $N$ . A Boolean response is obtained by thresholding the absolute difference  $|N^* - N| = \gamma$ . If  $\gamma \leq \tau$ ,  $A^*$  is authentic, otherwise  $A^*$  is declared a fake, as it is judged that modifications to  $A^*$  have

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altered the ill-conditioned matrix  $B^* = A^* \tilde{W}$  such that the error in solution  $N^*$  to expected solution  $N$  exceeds tolerance threshold  $\tau$ .

- 5 It will be clear to a person skilled in the art that whilst  $\hat{s}_r(A)$  and  $\hat{s}_t(W)$  are the preferred singular values to be replaced, an embodiment of the present invention may replace a singular value other than  $\hat{s}_r(A)$  or  $\hat{s}_t(W)$ , although for  $\hat{s}_r(A)$  this is likely to increase distortion in the
- 10 watermarked block  $\hat{A}$ .

- It will also be clear to a person skilled in the art that tractable linear and non-linear problems other than the minimisation problem of Eq. 8 and 12 that involve an ill-
- 15 conditioned operator may be amenable to the methods described herein.

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## 3. Supplementary information

For the purposes of clarity, the following provides detailed proofs of the ability to find an ill-conditioned operator  $B$  for a given  $A$ , and the ability to find a value  $\bar{s}_r(A) \in [H_0, H_1]$ . It also provides a discussion of the possible values of key  $N$ .

To prove the ill-conditioning of  $B$ , let  $A$  and  $W$  be two square matrices of the same dimension and  $s_k(A)$ ,  $s_k(W)$  their  $k$ -th singular values, respectively. Then,  
 $s_{i+j-1}(AW) \leq s_i(A)s_j(W)$ , for all integers  $i, j$ . (For the proof of this result, see A. Pietsch, *Eigenvalues and s-Numbers*, Cambridge University Press, 1997, Proposition 2.3.12.)

Next, let the smallest singular values of  $B=AW$  and  $W$  be  $s_r(B)$  and  $s_t(W)$ , respectively. Then

$$s_r(B) \leq s_{r-t+1}(A) \cdot s_t(W) = \varepsilon \cdot s_{r-t+1}(A) \text{ for } t \leq r. \quad \text{Eq. 13}$$

This follows directly from the previous result by setting  $i=r-t+1$  and  $j=t$ .

Since  $\varepsilon$  is chosen to be very small, the inequality Eq. 13 guarantees that the smallest singular value of  $B$  is also tiny and therefore extremely ill-conditioned.

Usually, the matrices  $A$  and  $W$  have full rank, i.e.,  $t = r$ . However, it is possible to build counterexamples with  $t > r$ . Even in such unusual situations Eq. 13 can be applied by setting  $s_k(W) = 0$  for all  $k > r$ . Observe that

because  $W$  is artificially constructed, there is nothing to prevents the required values being set to zero. As a

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consequence the condition  $t \leq r$  in Eq. 13 can be assumed in any case.

In order to prove the existence of  $\bar{s}_x(A) \in [H_0, H_1]$ ,  
 5 minimizing the expression Eq. 10 for a fixed value  $N$ ,  
 consider the real valued functions  $h(z) : [H_0, H_1] \rightarrow \mathbb{R}^+$ , and  
 $g(z) : [H_0, H_1] \rightarrow \mathbb{R}^+$  defined as  $h(z) = s_r(B)$  and

$$g(z) = \min_{x \in \mathbb{R}^p} \|B^*(z)x - b\|_2^2. \quad \text{Eq. 14}$$

$h(z)$  can be written as  $h(z) = s_r(A(z)\tilde{W}) = (h_1 \circ h_2)(z)$ , with  
 10  $h_1(z) = s_r(B(z))$  and  $h_2(z) = A(z)\tilde{W}$ . The two functions  $h_1$  and  $h_2$   
 are continuous in the interval  $[H_0, H_1]$ . Hence,  $h(z)$  is also  
 continuous in  $[H_0, H_1]$ . The continuity of  $h(z)$  can now be  
 used to prove that  $g(z)$  is continuous in  $[H_0, H_1]$ . Using Eq.  
 2 it is straightforward to derive the following expression:

$$15 \quad g(z) = \sum_{i=1}^n \left( u_{\hat{R}_i(z)}^T b / s_i(B(z)) \right)^2. \quad \text{Eq. 15}$$

Thus,  $g(z)$  is the sum of quotients of continuous functions.  
 Therefore,  $g(z)$  is also continuous in  $[H_0, H_1]$ .

Now, consider  $h_{\max} = \max(g(z))$  and  $h_{\min} = \min(g(z))$ . If  
 20  $N \in [g(h_{\max}), g(h_{\min})]$  then it exists  $\bar{z} \in [H_0, H_1]$  such that  
 $g(\bar{z}) = N$ . This follows from the continuity of  $g(z)$  in  $[H_0, H_1]$   
 and the mean-value theorem of continuous functions.

The above considerations illustrate the effectiveness and  
 25 feasibility of the proposed invention. The underlying  
 operator of Eq. 8 can be made extremely ill-conditioned  
 while the norm of its solution is kept equal to  $N$ .

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Furthermore, by selecting  $\hat{s}_r(A) \in [H_0, H_1]$  the distortion on the original image remains below the input parameter  $\delta$ .

However, this last property constrains the variation of  $\hat{s}_r(A)$  to a very small interval. Since  $\hat{s}_r(A)$  depends on  $N$ , an important question arises of how the small interval  $[H_0, H_1]$  constrains the set of feasible values  $N$ .

Since  $N$  is a secret key it is desirable that it is extremely difficult to estimate. Obviously the smaller the set of feasible values for  $N$ , the easier it is to estimate  $N$  and so mount a successful attack. This concern is addressed below.

Fortunately, the range of values that can be used for  $N$  is large, making difficult for an attacker to estimate it. Since the distortion introduced by the watermark can be strictly controlled by the distortion coefficient  $\delta$ , this coefficient defines the feasibility interval  $[H_0, H_1]$ .

Clearly, this interval is very small. Its maximum length does not exceed  $2\delta$  and according to the considerations above it defines the range of permissible values for  $N \in [g(h \max), g(h \min)]$ . Since  $N$  should be a large number to improve the security of the proposed algorithm, it is also important to show that the interval of permissible values of  $N$  is also very large. Variations of  $z \in [H_0, H_1]$  are reflected in the variations of the smallest singular value of  $B$ . According to Eq. 13 the smallest singular value of  $B$  is very close to  $\varepsilon$ . This fact can be used to find an estimate for the interval  $[g(h \max), g(h \min)]$ . For this we

consider the hyperbola  $p(z) = C + D / y^2$  with  $C = \sum_{i=1}^{r-1} (u_{B_i}^T b / s_i(B))^2$



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and  $D = (u_{\text{ex}}^T b)^2$ . Since the variation of  $z \in [H_0, H_1]$  determines the variation of  $p(z)$ , this gives the range for possible values  $N$ . Observe that changes in  $z$  also affect  $C$  and  $D$ , but actually the smallest singular value of  $B$  is the leading term determining the behaviour of  $p(z)$ . Clearly,  $p(z) \rightarrow \infty$  if  $z \rightarrow 0$ . Furthermore,  $p$  maps tiny intervals very close to zero into very large intervals. For instance, if  $\varepsilon = 10^{-16}$  and  $\delta = 10^{-2}$ , then  $z$  will approximately vary between the machine precision  $\text{eps}$ , e.g.,  $10^{-32}$ , and  $10^{-2}$ . In this case  $[g(h \text{ max}), g(h \text{ min})] \approx [10^2, 10^{32}]$ . As a consequence, for this particular example  $N$  could be selected from the interval  $N \in [10^2, 10^{32}]$ . These arguments show that the range of permissible values of  $N$  is huge and it would be extremely hard for an attacker to estimate  $N$ .

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## Claims

1. A method of fragile watermarking, characterised by the step of generating at least a first ill-conditioned operator, said ill-conditioned operator being related to values extracted from an image or portion thereof A.
2. A method of fragile watermarking according to claim 1, comprising the step of replacing a non-zero singular value of a singular value matrix  $S_A$  of an image or portion thereof A, with a solution to a linear equation comprising the ill-conditioned operator.
3. A method of fragile watermarking according to claim 2, wherein the non-zero singular value to be replaced is the smallest non-zero singular value  $S_r(A)$  in a singular value matrix  $S_A$  of rank  $r$ .
4. A method of fragile watermarking according to any one of the preceding claims, wherein a non-zero singular value of a singular value matrix  $S_W$  of a watermark pattern or portion thereof  $W$  is replaced, such that said replacement increases the condition number of the singular value matrix  $S_W$  of the watermark pattern or portion thereof  $W$ .
5. A method of fragile watermarking according to claim 4, wherein the non-zero singular value to be replaced is the smallest non-zero singular value  $S_t(W)$  in a singular value matrix  $S_W$  of rank  $t$ .
6. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating a replacement non-zero singular value of singular value

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matrix  $S_w$  of a watermark or portion thereof  $W$  comprises calculating substantially the following equation part:

$$s_r(W) = \varepsilon,$$

with all terms as defined herein.

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7. A method of fragile watermarking according to any one of the preceding claims, wherein the step of generating at least a first ill-conditioned operator comprises calculating substantially the following equation part:

$$10 \quad B = \tilde{A}\tilde{W},$$

where  $\tilde{W}$  is substantially constructed according to  $\tilde{W} = U_s \hat{S}_s V_s^T$ ,  $\hat{S}_s$  comprising at least one altered singular value  $s_r(W) = \varepsilon$ , and such that  $B$  forms a parametric family of matrices  $B(\hat{S}_s) = \tilde{A}(\hat{S}_s)\tilde{W}$  for possible values of  $\hat{S}_s(A)$ , with  
15 all remaining terms as defined herein.

8. A method of fragile watermarking according to claim 13, wherein  $s_r(A)$  is determined by an  $L_2$ -norm solution of the least squares problem  $\min_{x \in \mathbb{R}^p} \|Bx - b\|_2^2$  to equal the  
20 square of a predefined key  $N$  of predetermined value, with all terms as defined herein.

9. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating  
25 the replacement non-zero singular value of singular value matrix  $A$  comprises calculating substantially the following equation part:

$$\min_{\hat{S}_r(A)} \left\{ \sum_{i=1}^q (u_{R_i}^T b / s_i(B(\hat{S}_r)))^2 - N^2 \right\},$$

with all terms as defined herein.

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10. A method of fragile watermarking according to claim 9, wherein  $\hat{s}_r(A)$  further satisfies

$$\hat{s}_r(A) = \bar{s}_r(A) \in [\max(\epsilon p s_r, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1], \text{ such that}$$

the step of calculating the replacement non-zero singular

5 value comprises calculating substantially the following equation part:

$$\hat{s}_r \in [H_0, H_1] \left\{ \sum_{i=1}^q (u_{r_i}^T b / s_i(B(\hat{s}_r)))^2 - N^2 \right\},$$

with all terms as defined herein.

10 11. A method of fragile watermarking according to any one of claims 8 to 10, wherein vector  $b$  is related to at least a first parameter derived from a portion of an image  $I$  other than  $A$ .

15 12. A method of fragile watermarking according to claim 11, wherein for a sequential watermarking process comprising the watermarking of portion  $A^{(k)}$  after the watermarking of portion  $A^{(k-1)}$ ,  $k=1, \dots, L$  of  $L$  portions, then the step of calculating  $b^{(k)}$  for portion  $A^{(k)}$  comprises  
20 calculating substantially the following equation part:

$$b^{(k)} = \begin{cases} A^{(k)} Z^{(k)} & \text{for } k = 1 \\ A^{(k-1)} Z^{(k)} & \text{else} \end{cases}$$

where  $Z^{(k)}$  is a pseudo-random binary vector.

13. A method of fragile watermarking according to any one  
25 of the preceding claims, wherein the step of calculating the watermarked image or portion thereof  $\hat{A}$  comprises calculating substantially the following equation part:

$$\hat{A} = U_A \hat{S}_A V_A^T$$

where  $\hat{S}_A$  comprises at least one replaced singular value,

30 with all remaining terms as defined herein.

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14. A method of fragile watermarking according to any one of the preceding claims, wherein a watermark pattern or portion thereof  $W$  is generated by a pseudo-random generator seeded by a key  $K$  of predetermined value.

15. A method of fragile watermarking according to claim 14, wherein the values of key  $K$  and key  $N$  are related.

16. A method of fragile watermarking according to either one of claims 14 and 15, wherein the a watermark pattern or portion thereof  $W$  is generated by a pseudo-random generator seeded by a key  $K$  of predetermined value, combined with either a single or repeated instance of a logo.

15

17. A method of fragile watermarking according to any one of the preceding claims, comprising the following steps;

- i. generating a  $K$ -dependent watermark pattern  $W$  from  $\Omega$ , or recalling a pre-existing one;
- ii. constructing a parametric family of matrices  $B(\hat{s}_r)$ ;
- iii. estimating a unique parameter  $\bar{s}_r(A)$ , that minimizes the expression

$$\min_{\hat{s}_r} \left\{ \sum_{i=1}^q (u_{i,r}^T b / s_{i,r}(B(\hat{s}_r)))^2 - N^2 \right\}; \text{ and}$$

- iv. estimating the watermarked block  $\hat{A} = U_A \hat{S}_A V_A^T$  by setting  $\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{s}_r(A))$ ,

25

with all terms as defined herein.

18. A method of fragile watermarking according to any one of claims 1 to 16, comprising the following steps;

- i. generating a  $K$ -dependent watermark pattern  $W$  from  $\Omega$ , or recalling a pre-existing one;

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ii. constructing a parametric family of matrices  $B(\hat{s}_r)$ ;

iii. estimating a unique parameter

$\bar{s}_r(A) \in [\max(\epsilon s, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$ , that minimizes the expression:

$$\min_{\hat{s}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^q (u_{D_i}^T b / s_i(B(\hat{s}_r)))^2 - N^2 \right\}; \text{ and}$$

iv. estimating the watermarked block  $\tilde{A} = U_A \hat{S}_A V_A^T$  by

setting  $\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{s}_r(A))$ ,

with all terms as defined herein.

10 19. A method of verifying a fragile watermark, characterised by the step of generating at least a first ill-conditioned operator, said ill-conditioned operator being related to values extracted from a received image or portion thereof  $A^*$ .

15

20. A method of verifying a fragile watermark according to claim 19, characterised by the step of calculating a solution to the least squares problem  $\min_{x \in \mathbb{R}^p} \|B^* x - b\|_2^2$  where  $B^* = A^* \hat{W}$ , with all terms as defined herein.

20

21. A method of verifying a fragile watermark according to either one of claims 19 and 20, wherein a positive square-root  $N^*$  of the  $L_2$ -norm solution of the least squares

problem  $\min_{x \in \mathbb{R}^p} \|B^* x - b\|_2^2$  is compared with key  $N$ ; and

25

the received image or portion thereof  $A^*$  comprising the fragile watermark is declared authentic if  $|N^* - N| \leq \tau$ , where  $\tau$  is a threshold value.

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22. A method of verifying a fragile watermark according to any one of claims 19 to 21, wherein the step of calculating value  $N^*$  comprises calculating substantially the following equation part:

$$(N^*)^2 = \sum_{i=1}^n \left( u_{B_i}^T b / s_i(B^*) \right)^2,$$

with all terms as defined herein;

$N^*$  is compared with key  $N$ ; and

the received image or portion thereof  $A^*$  comprising the fragile watermark is declared authentic if  $|N^* - N| \leq \tau$ , where  $\tau$  is a threshold value.

23. Apparatus for fragile watermarking of an image in accordance with a method of any one of claims 1 to 18, and comprising;

generating means for generating at least a first ill-conditioned operator, said ill-conditioned operator being related to values extracted from an image or portion thereof  $A$ .

20

24. Apparatus for validating a fragile watermarked image in accordance with a method of any one of claims 19 to 22, and comprising;

generating means for generating at least a first ill-conditioned operator, said ill-conditioned operator being related to values extracted from a received image or portion thereof  $A^*$ .

25. A method of fragile watermarking according to claim 1 and substantially as hereinbefore described with reference to the accompanying drawings.

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26. A method of verifying a fragile watermark according to claim 19 and substantially as hereinbefore described with reference to the accompanying drawings.



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Abstract

Method and Apparatus for Fragile Watermarking

5 A method of fragile watermarking is characterised by the  
step of generating at least a first ill-conditioned  
operator, said ill-conditioned operator being related to  
values extracted from an image or portion thereof A.

10 FIG. 1

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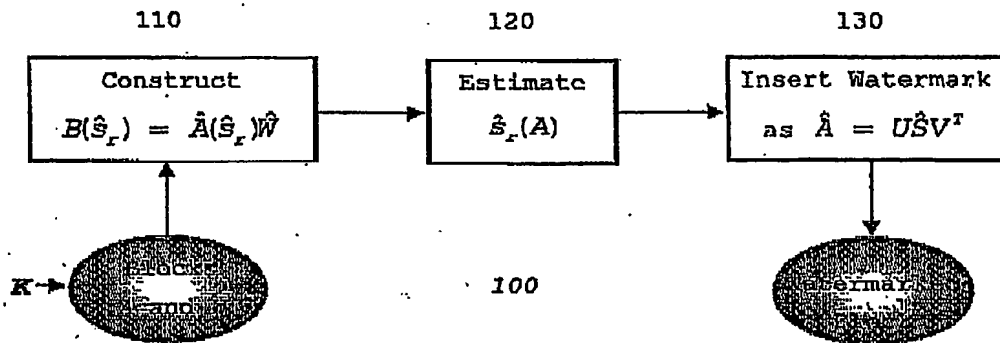


Figure 1

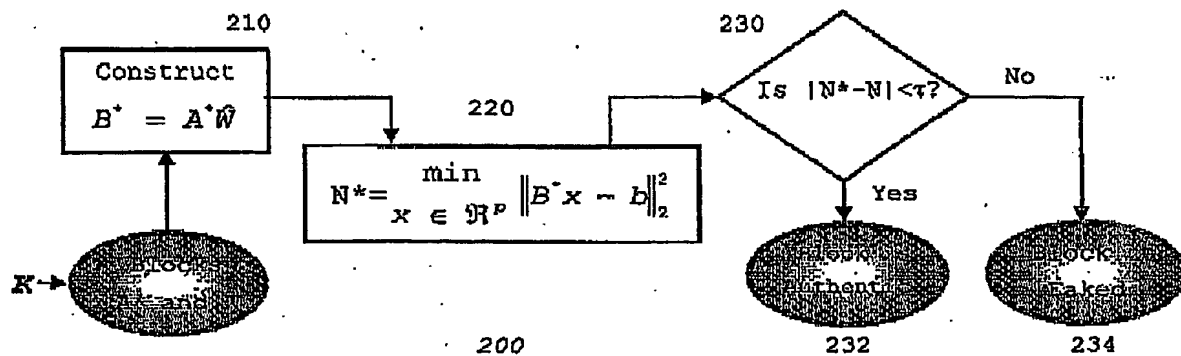


Figure 2

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